A Visual Flow Study of Mixed Convection in the Entry Region of a Vertical Internally Heated Annulus

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SUMMARY

The velocity distribution in laminar upward flow of water ($Pr \simeq 7{\cdot}25$) in the entry of a vertical internally heated annulus (radius ratio 4:1) has been determined by visual observation. Photographic measurements have been made of the motion of hydrogen bubble clusters, which were generated by a carefully controlled process of electrolysis, to assess the effects of free convection effects on the forced flow.

For heat fluxes up to 2500 W/m^2 and at a Reynolds number of approximately 450, local heat transfer coefficients have been obtained in a length of about 23 equivalent diameters. Heat transfer rate in the immediate entry was found to be insensitive to change in heat flux over the range of variables considered. As the distance downstream increased, the heat transfer rate was found to be dependent on the heat flux.

NOTATION

$\alpha(= d_0/d_1)$	radius ratio
β	coefficient of volumetric expansion
c_p	specific heat at constant pressure
$d, d_e(= d_0 - d_1)$	diameter, equivalent diameter
g	acceleration due to gravity
$h(= q_w/(T_w - T_\infty))$	local heat transfer coefficient
$h_b(= q_w/(T_w - T_b))$	local bulk heat transfer coefficient
k	thermal conductivity of fluid
$\mu, \nu(= \mu/\rho)$	absolute and kinematic viscosities
p	pressure
q	heat flux
ρ	fluid density
T	temperature
$\theta(= (T - T_i)/(T_w - T_i))$	dimensionless temperature
u, v	axial and radial component of velocity
z, r	axial and radial co-ordinates
$z^+(= z/d_e)$, $\bar{z}(= z^+/RePr)$	dimensionless axial distances
$R(= (r - r_1)/(r_0 - r_1))$, $r^+(= r/d_e)$	dimensionless radii
$Nu(= hd_e/k)$, $Nu_b(= h_b d_e/k)$	Nusselt numbers
$Pr(= c_p \mu/k)$, $Pe(= RePr)$	Prandtl number, Peclet number
$Re(= u_m d_e/\nu)$	Reynolds number
$Gr(= g\beta d_e^3(T_w - T_\infty)/\nu^2)$	Grashof number
$Ra(= GrPr)$	Rayleigh number

Suffixes

b, m	bulk, average values
i	inlet or pertaining to inlet temperature
1, o, w	inner wall, outer wall, wall
∞	value outside boundary layers

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INTRODUCTION

The most general case of low speed non-dissipative heat transfer in the presence of a body force field, is that in which the resultant velocity field is a consequence of the combined effects of internal buoyancy and externally impressed forces. The range of problems in this so-called mixed convection situation extends from the extremes of free convection on the one hand, when motion results from buoyancy effects alone, to the forced convection regime on the other, when motion is produced by external forces and the buoyancy effects are vanishingly small. The range is a continuous one, but it is convenient for the engineer to consider a tripartite subdivision into free, mixed, and forced convection following Metais and Eckert, (**1**), dependent on the values of the pertinent parameters. The range of problems in this field of study may be further subdivided according to the geometry, the direction of the force field relative to the main motion, and the thermal boundary conditions. With the advent of highly rated heat transfer surfaces, numerous situations arise in the mixed convection regime. Of special interest, in the nuclear engineering field, for example, is the case of upward flow in an internally heated vertical annulus in the region of the entry where the velocity is initially uniform, and this is the subject of study in the present work.

In view of the increasing importance of the mixed convection regime, considerable effort has been made particularly during the last two decades to gain some insight into the mechanisms involved and to obtain reliable working correlations. From the mathematician's point of view, mixed convection may be regarded as one of the mechanisms with the other superimposed, and so perturbation solutions have found favour with a number of authors. Sitharamarao (**2**), who has reviewed much of the recent experimental and theoretical work in this field, employed this technique following Sparrow and Gregg (**3**), Eshghy (**4**), and Szewczyk (**5**), and his theoretical results for a predominantly forced ducted flow are reported by Sitharamarao and Barrow (**6**). An alternative method of solution is an iterative solution of the finite difference equations after Sherwin and Wallis (**7**) whose work is particularly relevant here because it pertains to the same type of geometry and takes cognizance of the existence of appreciable radial velocity in a developing flow.

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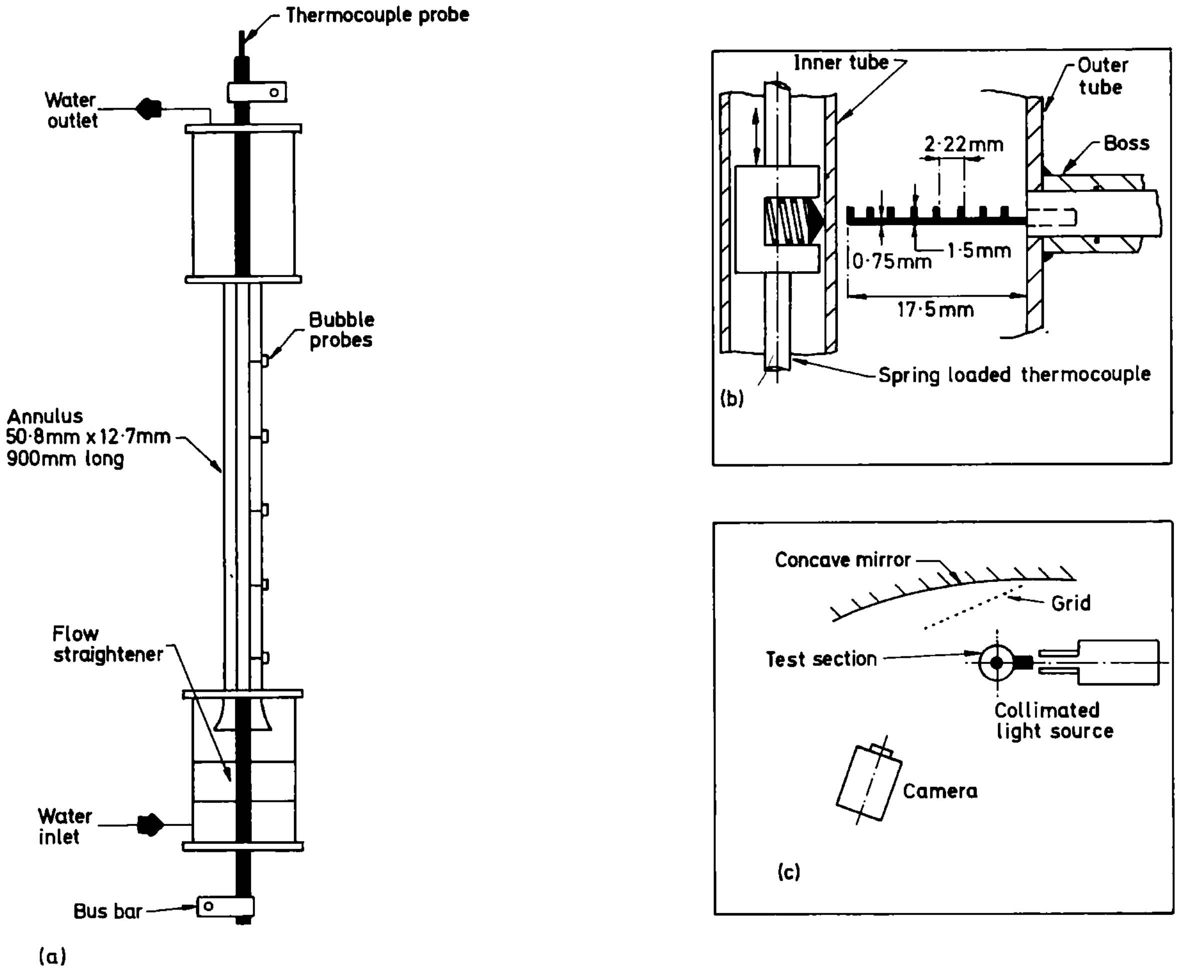


Fig. 1. Schematic arrangement of test section, and details of the thermocouple and the bubble probes and the technique used for recording visualization of the flow. (a) Test section (not to scale), (b) details of thermocouple and the bubble probes which are made from brass shim 0·05 mm thick, (c) an arrangement used to photograph hydrogen bubbles

Perhaps not surprisingly, experimental studies of the details of the flow in combined convection are less common although there has been fairly extensive investigation of the overall heat transfer effects. Mixed convection in vertical tubes has been studied by Hallman (**8**) and Scheele and Hanratty (**9**), for example, while the annulus was investigated by Sitharamarao (**2**) and Sherwin (**10**).

The detailed measurement of the flow field is clearly difficult although attempts have been made using dye filaments. The familiar dye-filament technique has a number of serious disadvantages and is not convenient when the velocities at a multiplicity of points are required to be measured simultaneously. An improved technique for measuring velocities in free convection gas flows is that reported by Eichhorn (**11**), who photographed the trajectories of dust particles (6·6 μm diameter), but this cannot be used in ducted liquid flows. In the work reported in this paper, the velocity field over a large area was measured using the hydrogen bubble technique which is described by Schraub, Kline, Runstalder, Henry, and Littell (**12**). With this method, the whole velocity field can be calculated from photographic records of the displacement of bubbles which are generated periodically at cathodic probes located in the region of interest. In the following sections, the experimental technique which has been developed is outlined and the analysis of the data and their interpretation are described briefly. For completeness, some measurements of local heat transfer coefficient are included and compared with the findings of other investigations.

APPARATUS AND EXPERIMENTAL PROCEDURE

A diagrammatic layout of the test section of the experimental apparatus is shown in Fig. 1(a). The test section consisted of a stainless steel tube, 12·7 mm dia. located centrally in a 50·8 mm bore Perspex tube, 900 mm long, fitted with inlet and outlet water reservoirs. Axisymmetric flow and a uniform velocity at inlet to the test length were produced by a well-designed fibreglass bell mouth and flow straighteners at the inlet, while the whole apparatus was suspended freely in the vertical position with a view to obtaining good concentricity of the core and containing tube.

The open water circuit included a constant head tank, pump, rotameter, and preheater in order that prescribed inlet conditions could be achieved. Accurate measurement of the flowrate was made using a graduated flask and fluid temperatures were recorded by thermocouples installed at appropriate points in the circuit. Local bulk temperatures were calculated using an energy balance.

The stainless steel tube was heated electrically using a transformer, its inner wall temperature at any axial location being measured by spring loaded thermocouple probes, Fig. 1(b), which could be inserted in the bore of the tube from the top. Earlier tests in a separate apparatus showed that the readings obtained from a spring loaded thermocouple probe were in good agreement with those obtained from a thermocouple soldered to the surface of a tube.

As mentioned previously, radial and axial velocities over the whole field were determined by photographing hydrogen bubbles which were generated by electrolysis at a number of axially-located radial probes.† A detail of one such probe is shown in Fig. 1(b) from which it can be seen that bubbles could be generated at known radial positions. Using a pulse generator, described by Pearson and Baxter (**13**), bubbles were shed from the probes simultaneously. (The pulsed voltage, frequency, and duration could be varied depending on flow conditions.) A technique of indirect photography was employed. By illuminating the bubbles with a collimated tungsten-halogen light source on the right-hand side, and using a large mirror which was concave in the horizontal, the enlarged images of the bubbles were photographed. The arrangement of the photographic equipment is sketched in Fig. 1(c), which also shows a reference measuring grid of known mesh size. The image of the grid was superimposed on the bubble picture. The substantial enlargement which the concave mirror produced in the radial (or horizontal) direction facilitated easier and more accurate measurement of distances in this direction. The origins of the bubble clusters were known from the geometry and location of the probes. Knowing the frequency of generation of the bubbles and the axial and radial distances of the bubble clusters measured from their sources of origin, the local velocity vectors in the flow field could be determined.

The major part of the experimental study pertained to the steady state condition and all the experimental data presented in a following section refer to this situation. For a given flowrate and power input to the core tube, the relevant experimental data were recorded and the corresponding bubble pattern photographed. Some tests were carried out in the transient state, however, the electric power to the stainless steel tube being increased in one step to observe the ensuing flow response. A number of these tests were recorded on cine-film and convincingly demonstrate the changing flow pattern brought about by the free convection effect on the developing flow. Heat transfer coefficients, as defined later, were evaluated from the experimental power and temperature measurements, due recognition being given to heat conduction in the wall of the heated tube. These results and the visual flow observations are presented in the Section, 'Experimental Results and Discussion'. The following section is devoted to some considerations of the basic equations for the mixed convection problem in ducted flow: it is intended to clarify the derivation of the parameters which control the flow and heat transfer in the present problem.

† In addition to still photographs, a short cine-film was made.

THEORETICAL CONSIDERATIONS

In seeking the pertinent dimensionless parameters in combined free and forced laminar convection in an annulus, reference is made to the axial momentum equation and the energy equation.

Respectively

$$\rho\left(u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial z}+\mu\nabla^2 u-\rho g \tag{1}$$

and

$$u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}=\frac{k}{c_p\rho}\nabla^2 T \tag{2}$$

Since in the entrance flow, Bernoulli's equation is valid outside the boundary layers, then eq. (1) may be further developed to read

$$u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}=u_\infty\frac{\partial u_\infty}{\partial z}+\nu\nabla^2 u+g\beta(T-T_\infty) \tag{3}$$

with the usual assumption that $\rho=\rho_\infty$, and where the coefficient of expansion $\beta=-1/\rho(\partial\rho/\rho T)_p$.

By applying the familiar ideas of similarity to eq. (3), (see for example Eckert and Drake (**14**), it can be shown that the velocity field for given boundary conditions is a function of the dimensionless co-ordinates z^+ and r^+, and the parameters Re and Gr.

Using eq. (2) in like manner, the dimensionless temperature θ is found to be a function of z^+, r^+, and Pr.

If a local heat transfer coefficient, Nu, is defined on the basis of the temperature difference between the local wall temperature and the inlet temperature, which is to the advantage of the engineer, then

$$Nu=f(z^+,\alpha,Re,Gr,Pr) \tag{4}$$

where it is necessary to include the radius ratio, α, because z and d_e alone do not completely define the geometry. A further restriction to the use of a unique form of eq. (4) is that the boundary conditions for velocity and dimensionless temperature must be the same.

It is to be emphasized that provided the parameters are as defined (see Notation), then eq. (4) is exact with the proviso that the basic equations (1) and (2) accurately describe the flow and heat transfer.

Other authors have found occasion to use the parameters Nu and Gr defined in different ways which has tended to confuse rather than clarify the problem. One such technique is to employ the local wall to local fluid bulk temperature difference, and for the purpose of comparison with some theoretical work pertaining to the results of the present mixed convection study, this method will be adopted here. The experimental results for the heat transfer coefficients are presented in the next section together with the observations on the velocity distribution which plays an all important role in the convection process.

EXPERIMENTAL RESULTS AND DISCUSSION

The main purpose of this investigation was to observe the fluid flow in detail and to make measurements of the axial and radial components of velocity over the whole field with a view to explaining the dependence of heat transfer coefficient on the heating rate.

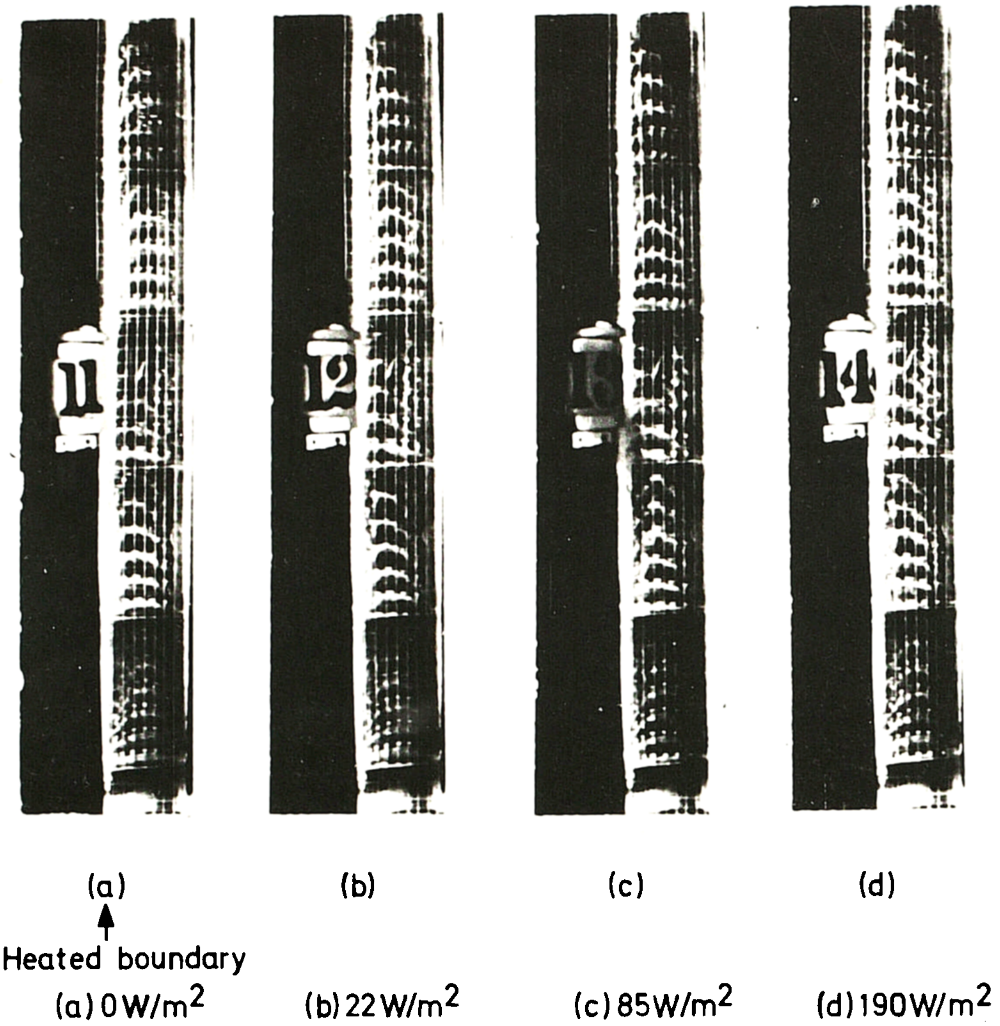


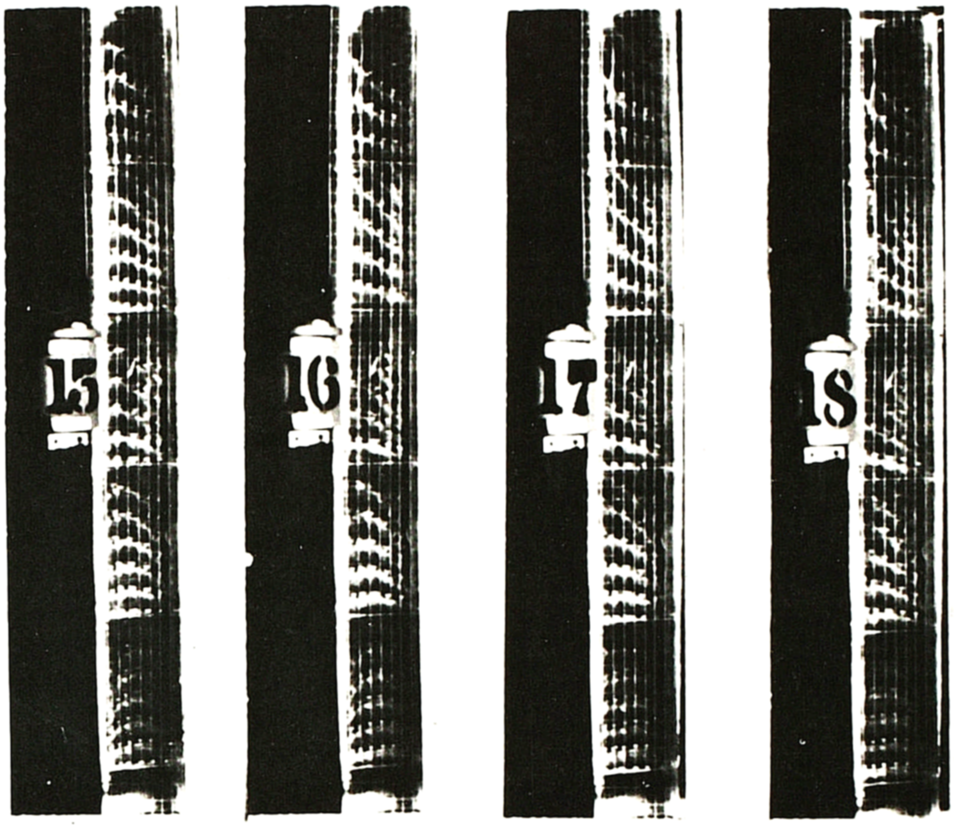
Fig. 2. Typical bubble patterns in upward flow in the annulus for various values of heat flux ($Re = 445$) (a) 0 W/m^2, (b) 22 W/m^2, (c) 85 W/m^2, (d) 190 W/m^2, (e) 315 W/m^2, (f) 760 W/m^2, (g) 1450 W/m^2, (h) 2340 W/m^2

Using the technique described earlier, photographs of the bubble patterns were taken for a given flowrate with various heat inputs. Figure 2 shows a typical set of data, although enlargements ($\times 3\frac{1}{2}$) of these prints were used for the actual measurement of displacements for the purpose of determining the velocity vector. The data shown in Fig. 2 refer to the case of $Re = 445$, and a bubble pulse to pulse interval of 1·4 sec. with the average wall heat fluxes as indicated.

Processing of the data involved curve fitting of the information obtained by direct measurements from the photographs, a computer program for the K.D.F.9 computer at Liverpool University being used for this purpose. The program incorporated procedures for the evaluation of the axial and radial components of velocity throughout the flow field and for the evaluation of the integrated mass flow at various axial positions. An essential condition of the velocity component distribution is that continuity must be satisfied and the latter feature of the computational work served as a check on this requirement. Additional procedures were included to translate the data associated with heat transfer coefficient and to evaluate local Nusselt and Grashof numbers.

The most noticeable feature in the series of photographs is the marked increase in the velocity near the heated boundary as the heating rate increases, the increase being particularly evident at some distance from the entry. In Fig. 2(a), corresponding to zero heat flux, the velocity profiles appear similar to those for adiabatic laminar flow, while as the heat flux increases (Fig. 2(b) to Fig. 2(h)), the onset of buoyant motion near the heated wall may be seen. The tendency towards a possible flow reversal near the outer wall is to be observed in Fig. 2(h) corresponding to the largest heat flux but the occurrence of this phenomenon was not established with any certainty. There was noticeably little change in the bubble pattern over the first few diameters of the flow length, possible explanation for which will be given when the local heat transfer coefficients are considered later. Clearly some distance is necessary before the flow responds to the effects of buoyancy indicating a predominance of forced convection in the immediate entry.

The axial velocity components at two axial stations for adiabatic flow ($q_w = 0$, Fig. 2(a)) and heated flow ($q_w = 2340$ W/m^2, Fig. 2(h)) are presented in Fig. 3(a) and (b). The development of the profile in diabatic flow is detectable; the profile becomes less square as z increases with the radius of maximum velocity occurring closer to the inner boundary. The theoretical fully de-

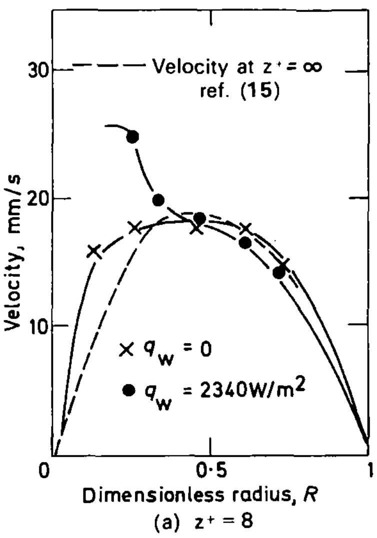


(e) (f) (g) (h)

(e) 315 W/m^2 (f) 760 W/m^2 (g) 1450 W/m^2 (h) 2340 W/m^2

veloped profile according to Heaton, Reynolds, and Kays (**15**) has been included and shows only fair agreement with the trend of the present results. In view of the nature of the determination of these data, however, the result is encouraging and, with refinement of the computational technique, considerable improvement can be anticipated. The heated flow velocity profiles emphasize what has been said earlier concerning the increase in velocity near the heated surface and the progressive reduction at the larger radii. Again, continuity of mass must be maintained, and smaller reductions in velocity at the larger radii (where the contribution to mass flux is greatest) are compatible with the very substantial increases near the heated wall.

As previously stated, Nusselt numbers were computed for completeness and in order that a comparison might be made, those based on the local bulk temperature have been adopted. Figure 4 shows the results.



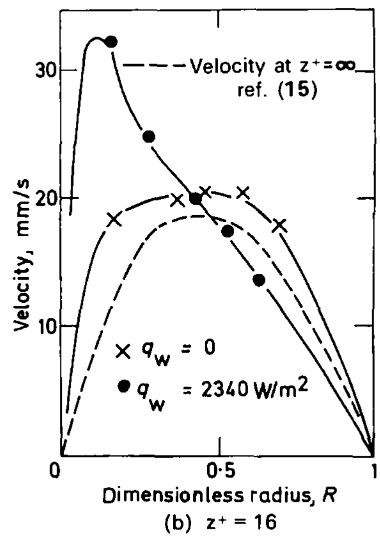


Fig. 3. Typical velocity profiles in adiabatic and heated flows

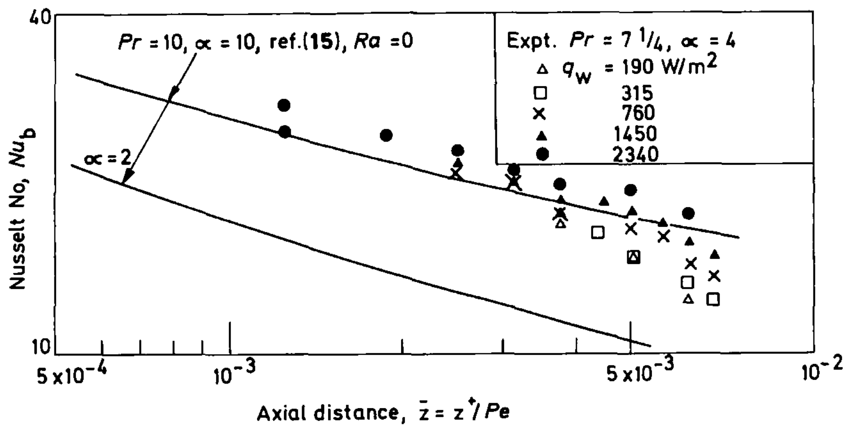


Fig. 4. Entry region heat transfer coefficients for water in mixed convection in an annulus

Included in this plot are the theoretical curves of Heaton, Reynolds, and Kays (**15**), for the two radius ratios 2 and 10 and for a Prandtl number equal to 10. These theoretical results refer to a flow with zero Rayleigh number and serve only as a guide to the accuracy of the present data. The plot convincingly shows that the dependency of heat transfer coefficient on heat input increases along the length of the duct, significant increases due to the free convection effect beginning to appear at about $\bar{z} = 4 \times 10^{-3}$. This finding is consistent with the visual flow observations in Fig. 2 and the quantitative results for the velocity field in Fig. 3 where it is to be observed that very marked distortion of the flow does not occur in the immediate vicinity of the entry. As explained by Hallman (**8**), the Nusselt numbers are dependent on the shape of the velocity profile, and so near the entry they will be unaffected by the heat transfer rate. The velocity profile in turn will not be distorted from its near square shape until a temperature field has been established to produce a radial density gradient. The present observations then substantiate Hallman's (**8**) conclusion concerning mixed convection in the thermal entry length, that a certain distance is required to bring about a distorted flow pattern. In other words, it might be said that forced convection predominates in the heat transfer mechanism in the initial part of the flow. At this stage, a quantitative measure of this distance is not suggested as more intensive investigation is required over a wider range of heat transfer and flow variables.

Radial velocity components were in evidence in the visual flow observation experiments and these data may be determined, if required, using the observed radial displacements and pulsation times.

The present investigation was concerned principally with effects in the immediate entry of the inlet to the duct and there was insufficient length for a pseudo-fully developed laminar flow to be established. Indeed such a situation could not have been achieved even in a longer duct since the photographs showed the onset of turbulence near the outlet ($z^+ \simeq 20$) at larger heat inputs. The criterion for the transition point in tube flow has been obtained by Hallman (**8**), who used a photoelectric record to sense temperature fluctuations in the wall. The present visual technique will afford a more direct method of determining this change and its behaviour with regard to unsteadiness.

The use of the hydrogen bubble method as employed here and in other flow problems invites the question concerning the error due to the relative motion between the bubble and its containing fluid. This relative velocity can be calculated for known bubble size, gas and fluid densities, and can be shown to be relatively small compared with absolute fluid velocities except in the regions very close to the walls. A check on this is the calculated integrated mass flowrate compared with the experimentally measured flowrate. Had there been any significant error in using the bubbles as markers this would have been immediately apparent. As it was, the integrated value was slightly larger than the measured value and could account for the discrepancy between the adiabatic profiles of Fig. 3 and the theoretical curves assuming the latter to be accurate. If necessary, this question of bubble buoyancy could be resolved by repeating the experiment with downward adiabatic flow and comparing the profiles with those in Fig. 3. It will be observed that in the present tests measurements of bubble velocities very close to the walls were avoided.

CONCLUSIONS

A technique using the hydrogen bubble method has been tested to measure fluid velocities in mixed convection in the upward flow of water in an annulus. At the same time, local heat transfer coefficients have been determined and their variation has been considered in the light of the results of visualization of the flow.

The technique shows promise both for the quantitative measurement of velocity components over a large flow area and for the observation of phenomena which are peculiar to the mixed regime of flow. The range of variables studied in the investigation was limited but sufficient to indicate trends and the dependency of entry-length heat transfer on the magnitude of the heat flux.

It has been found that free convection effects on an otherwise forced flow do not have an immediate influence on heat transfer and in the immediate vicinity of the start of the flow the forced convection solution will pertain. This result is consistent with the findings of earlier workers in this field of study, but further data will be required before an attempt at correlation can be made.

It is concluded that further inquiry by the technique and methods described in this report is warranted.

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